RADIATIVE VIEW FACTORS

VIEW FACTOR DEFINITION

The view factor F_{12} is the fraction of energy exiting an isothermal, opaque, and diffuse surface 1 (by emission or reflection), that directly impinges on surface 2 (to be absorbed, reflected, or transmitted). View factors depend only on geometry. Some view factors having an analytical expression are compiled below. We will use the subindices in F_{12} without a separator when only a few single view-factors are concerned, although more explicit versions, like $F_{1,2}$, or even better, $F_{1\to2}$, could be used.

From the above definition of view factors, we get the explicit geometrical dependence as follows. Consider two infinitesimal surface patches, dA_1 and dA_2 (Fig. 1), in arbitrary position and orientation, defined by their separation distance r_{12}, and their respective tilting relative to the line of centres, β_1 and β_2, with $0\leq\beta_1\leq\pi/2$ and $0\leq\beta_2\leq\pi/2$ (i.e. seeing each other). The expression for dF_{12} (we used the differential symbol 'd' to match infinitesimal orders of magnitude, since the fraction of the radiation from surface 1 that reaches surface 2 is proportional to dA_2), in terms of these geometrical parameters is as follows. The radiation power intercepted by surface dA_2 coming directly from a diffuse surface dA_1 is the product of its radiance $L_1=M_1/\pi$, times its perpendicular area $dA_{1\perp}$, times the solid angle subtended by dA_2, $d\Omega_{12}$; i.e. $d^2\Phi_{12}=L_1 dA_{1\perp} d\Omega_{12}=L_1(dA_1\cos(\beta_1))dA_2\cos(\beta_2)/r_{12}^2$. Thence:

$$dF_{12} \equiv \frac{d^2\Phi_{12}}{M_1 dA_1} = \frac{L_1 d\Omega_{12} dA_1 \cos(\beta_1)}{M_1 dA_1} = \frac{\cos(\beta_1)}{\pi} d\Omega_{12} = \frac{\cos(\beta_1)}{\pi}\frac{dA_2\cos(\beta_2)}{r_{12}^2} = \frac{\cos(\beta_1)\cos(\beta_2)}{\pi r_{12}^2} dA_2 \quad (1)$$

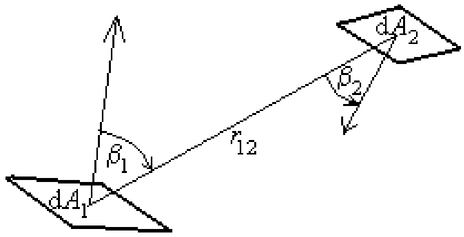


Fig. 1. Geometry for view-factor definition.

When finite surfaces are involved, computing view factors is just a problem of mathematical integration (not a trivial one, except in simple cases). Notice that the view factor from a patch dA_1 to a finite surface A_2, is just the sum of elementary terms, whereas for a finite source, A_1, the total view factor, being a fraction, is the average of the elementary terms, i.e. the view factor between finite surfaces A_1 and A_2 is:

$$F_{12} = \frac{1}{A_1}\int_{A_1}\left(\int_{A_2}\frac{\cos\beta_1\cos\beta_2}{\pi r_{12}^2} dA_2\right) dA_1 \quad (2)$$

Recall that the emitting surface (exiting, in general) must be isothermal, opaque, and Lambertian (a perfect diffuser for emission and reflection), and, to apply view-factor algebra, all surfaces must be isothermal, opaque, and Lambertian. Finally notice that F_{12} is proportional to A_2 but not to A_1.

View factor algebra

When considering all the surfaces under sight from a given one (let the enclosure have N different surfaces, all opaque, isothermal, and diffuse), several general relations can be established among the N^2 possible view factors F_{ij}, what is known as view factor algebra:

- Bounding. View factors are bounded to $0\leq F_{ij}\leq 1$ by definition (the view factor F_{ij} is the fraction of energy exiting surface i, that impinges on surface j).
- Closeness. Summing up all view factors from a given surface in an enclosure, including the possible self-view factor for concave surfaces, $\sum_j F_{ij} = 1$, because the same amount of radiation emitted by a surface must be absorbed.
- Reciprocity. Noticing from the above equation that $dA_i dF_{ij}=dA_j dF_{ji}=(\cos\beta_i\cos\beta_j/(\pi r_{ij}^2))dA_i dA_j$, it is deduced that $A_i F_{ij} = A_j F_{ji}$.
- Distribution. When two target surfaces (j and k) are considered at once, $F_{i,j+k} = F_{ij} + F_{ik}$, based on area additivity in the definition.

- Composition. Based on reciprocity and distribution, when two source areas are considered together, $F_{i+j,k}=\left(A_iF_{ik}+A_jF_{jk}\right)/\left(A_i+A_j\right)$.

One should stress the importance of properly identifying the surfaces at work; e.g. the area of a square plate of 1 m in side may be 1 m^2 or 2 m^2, depending on our considering one face or the two faces. Notice that the view factor from a plate 1 to a plate 2 is the same if we are considering only the frontal face of 2 or its two faces, but the view factor from a plate 1 to a plate 2 halves if we are considering the two faces of 1, relative to only taking its frontal face.

For an enclosure formed by N surfaces, there are N^2 view factors (each surface with all the others and itself). But only $N(N-1)/2$ of them are independent, since another $N(N-1)/2$ can be deduced from reciprocity relations, and N more by closeness relations. For instance, for a 3-surface enclosure, we can define 9 possible view factors, 3 of which must be found independently, another 3 can be obtained from $A_iF_{ij}=A_jF_{ji}$, and the remaining 3 by $\sum_j F_{ij}=1$.

View factors with two-dimensional objects

Consider two infinitesimal surface patches, dA_1 and dA_2, each one on an infinitesimal long parallel strip as shown in Fig. 2. The view factor dF_{12} is given by (1), where the distance between centres, r_{12}, and the angles β_1 and β_2 between the line of centres and the respective normal are depicted in the 3D view in Fig. 2a, but we want to put them in terms of the 2D parameters shown in Fig. 2b (the minimum distance $a=\sqrt{x^2+y^2}$, and the β_1 and β_2 angles when z=0, β_{10} and β_{20}), and the depth z of the dA_2 location. The relationship are: $r_{12}=\sqrt{x^2+y^2+z^2}=\sqrt{a^2+z^2}$, $\cos\beta_1=\cos\beta_{10}\cos\gamma$, with $\cos\beta_1=y/r_{12}=(y/a)(a/r_{12})$, $\cos\beta_{10}=y/a$, $\cos\gamma=a/r_{12}$, and $\cos\beta_2=\cos\beta_{20}\cos\gamma$, therefore, between the two patches:

$$\mathrm{d}F_{12}=\frac{\cos(\beta_1)\cos(\beta_2)}{\pi r_{12}^2}\mathrm{d}A_2=\frac{a^2\cos(\beta_{10})\cos(\beta_{20})}{\pi r_{12}^4}\mathrm{d}A_2=\frac{a^2\cos(\beta_{10})\cos(\beta_{20})}{\pi\left(a^2+z^2\right)^2}\mathrm{d}A_2 \qquad (3)$$

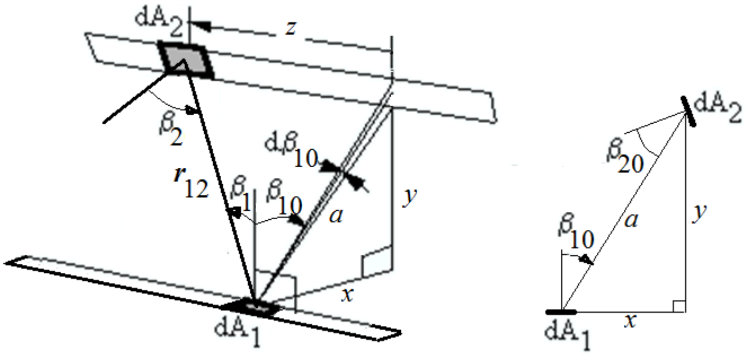


Fig. 2. Geometry for view-factor between two patches in parallel strips: a 3D sketch, b) profile view.

Expression (3) can be reformulated in many different ways; e.g. by setting d$^2A_2$=dwdz, where the 'd^2' notation is used to match differential orders and dw is the width of the strip, and using the relation ad$\beta_{10}=\cos\beta_{20}dw$. However, what we want is to compute the view factor from the patch dA_1 to the whole strip from $z=-\infty$ to $z=+\infty$, what is achieved by integration of (3) in z:

$$\mathrm{d}^2F_{12}=\frac{a^2\cos(\beta_{10})\cos(\beta_{20})}{\pi\left(a^2+z^2\right)^2}\mathrm{d}w\mathrm{d}z \quad\rightarrow\quad \mathrm{d}F_{12}=\int_{-\infty}^{\infty}\mathrm{d}^2F_{12}\mathrm{d}z=\frac{\cos(\beta_{10})\cos(\beta_{20})}{2a}\mathrm{d}w=\frac{\cos(\beta_{10})}{2}\mathrm{d}\beta_{10} \quad (4)$$

For instance, approximating differentials by small finite quantities, the fraction of radiation exiting a patch of A_1=1 cm^2, that impinges on a parallel and frontal strip ($\beta_{10}=\beta_{20}=0$) of width w=1 cm separated a distance a=1 m apart is $F_{12}=w/(2a)=0.01/(2\cdot 1)=0.005$, i.e. a 0.5 %. It is stressed again that the exponent in the differential operator 'd' is used for consistency in infinitesimal order.

Now we want to know the view factor dF_{12} from an infinite strip dA_1 (of area per unit length dw_1) to an infinite strip dA_2 (of area per unit length dw_2), with the geometry presented in Fig. 2. It is clear from the infinite extent of strip dA_2 that any patch $\mathrm{d}^2A_1=\mathrm{d}w_1\mathrm{d}z_1$ has the same view factor to the strip dA_2, so that the average coincides with this constant value and, consequently, the view factor between the two strips is precisely given by (4); i.e. following the example presented above, the fraction of radiation exiting a long strip of w_1=1 cm width, that impinges on a parallel and frontal strip ($\beta_{10}=\beta_{20}=0$) of width w_2=1 cm separated a distance a=1 m apart is $F_{12}=w_2/(2a)=0.01/(2\cdot 1)=0.005$, i.e. a 0.5 %.

Notice the difference in view factors between the two strips and the two patches in the same position as in Fig. 2b: using dA_1 and dA_2 in both cases, the latter (3D case) is given by the general expression (1), which takes the form $\mathrm{d}F_{12}=\cos\beta_{10}\cos\beta_{20}\mathrm{d}A_2/(\pi a^2)$, whereas in the two-strip case (2D), it is $\mathrm{d}F_{12}=\cos\beta_{10}\cos\beta_{20}\mathrm{d}A_2/(2a)$, where A_2 has now units of length (width of the strip).

Very-long triangular enclosure

Consider a long duct with the triangular cross section shown in Fig. 3. We may compute the view factor F_{12} from face 1 to face 2 (inside the duct) by double integration of the view factor from a strip of width dw_1 in L_1 to strip dw_2 in L_2; e.g. using de strip-to-strip view factor (4), the strip to finite band view factor is $F_{12}=\int\cos\beta_{10}\mathrm{d}\beta_{10}/2=(\sin\beta_{10\mathrm{end}}-\sin\beta_{10\mathrm{start}})/2$, where $\beta_{10\mathrm{start}}$ and $\beta_{10\mathrm{end}}$ are the angular start and end directions subtended by the finite band 2 from infinitesimal strip 1. Let see an example to be more explicit.

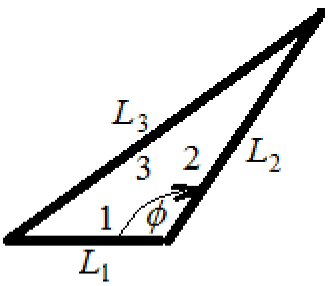


Fig. 3. Triangular enclosure.

<u>Example 1</u>. Find the view factor F_{12} from L_1 to L_2 for ϕ=90º in Fig. 3, i.e. between two long perpendicular strips touching, by integration of the case for infinitesimal strips (4).

Sol. Referring to Fig. 2b, the view factor from a generic infinitesimal strip 1 at x (from the edge), to the whole band at 2, becomes F_{12}=(sin$\beta_{10\text{end}}$−sin$\beta_{10\text{start}}$)/2=$(1-x/\sqrt{x^2+L_2^2})/2$, and, upon integration on x, we get the view factor from finite band 1 to finite band 2: $F_{12}=(1/L_1)\int(1-x/\sqrt{x^2+L_2^2})\mathrm{d}x/2=\left(L_1+L_2-\sqrt{L_1^2+L_2^2}\right)/(2L_1)$.

But it is not necessary to carry out integrations because all view factors in such a simple triangular enclosure can be found by simple application of the view-factor algebra presented above. To demonstrate it, we first establish the closure relation $\sum_j F_{ij}=1$ at each of the three nodes, noticing that for non-concave surfaces F_{ii}=0; then we multiply by their respective areas (in our case L_1, L_2, L_3, by unit depth length); next, we apply some reciprocity relations, and finally perform de combination of equations as stated below:

$$0+F_{12}+F_{13}=1 \quad\rightarrow\quad L_1F_{12}+L_1F_{13}=L_1 \tag{5}$$

$$F_{21}+0+F_{23}=1 \quad\rightarrow\quad L_2F_{21}+L_2F_{23}=L_2 \quad\rightarrow\quad L_1F_{12}+L_2F_{23}=L_2 \tag{6}$$

$$F_{31}+F_{32}+0=1 \quad\rightarrow\quad L_3F_{31}+L_3F_{32}=L_3 \quad\rightarrow\quad L_1F_{13}+L_2F_{23}=L_3 \tag{7}$$

$$(5)+(6)-(7) \quad\Rightarrow\quad 2L_1F_{12}=L_1+L_2-L_3 \quad\rightarrow\quad F_{12}=\frac{L_1+L_2-L_3}{2L_1} \tag{8}$$

We see how easy it is now to recover the result for perpendicular bands of width L_1 and L_2 solved in Example 1, $F_{12}=\left(L_1+L_2-\sqrt{L_1^2+L_2^2}\right)/(2L_1)$; e.g. the view factor between equal perpendicular bands is $F_{12}=\left(2-\sqrt{2}\right)/2$=0.293, i.e. 29 % of the energy diffusively outgoing a long strip will directly reach an equal strip perpendicular and hinged to the former, with the remaining 71 % being directed to the other side 3 (lost towards the environment if L_3 is just an opening).

Even though we have implicitly assumed straight-line cross-sections (Fig. 3), the result (8) applies to convex triangles too (we only required F_{ii}=0), using the real curvilinear lengths instead of the straight distances. As for concave bands, the best is to apply (8) to the imaginary straight-line triangle, and afterwards solve for the trivial enclosure of the real concave shape and its corresponding virtual straight-line.

<u>Example 2</u>. Find the view factor F_{12} between two long hemicylindrical strips touching with perpendicular faces (Fig. E2).

Sol. Let 1’ and 2’ be the imaginary planar faces of the hemicyliners. From Example 1 applied to equal perpendicular straight strips, $F_{1'2'}=\left(L_{1'}+L_{2'}-\sqrt{L_{1'}^2+L_{2'}^2}\right)/(2L_{1'})=\left(2-\sqrt{2}\right)/2$=0.293, where $L_{1'}=L_{2'}$ are the diametrical strips to L_1 and L_2 (dashed in Fig. E2), which we can take as unit length; the enclose is completed with $F_{1'3}=1-F_{1'2'}$=0,707. Let consider now the cylindrical surfaces; first, it is clear that the radiation arriving at 2’ (from 1’) will arrive at 2 also, i.e. $F_{1'2'}=F_{1'2}$; second, it is obvious that $F_{1'1}$=1, i.e. all radiation outgoing downwards from the planar strip 1’ will go to 1, and, by the reciprocity relation, $F_{11'}=A_{1'}F_{1'1}/A_1=1\cdot F_{1'1}/(\pi/2)=2/\pi$= 0.637, and hence F_{11}=1–$F_{11''}$=1–2/π=0,363. In summary, from the radiation cast by hemicylinder 1, a fraction F_{11}=36 % goes against the same concave surface, and, from the remaining $F_{11'}$=64 % that goes upwards, 29 % of it goes towards the right (surface 2), and the 71 % remaining impinged on surface 3; in consequence, F_{12}=$F_{11'}F_{1'2}$=0.637·0.293=$\left(2-\sqrt{2}\right)/\pi$ =0.186, F_{13}=$F_{11'}F_{1'3}$= 0.637·0.707=$\sqrt{2}/\pi$ =0.450, and F_{11}=1–2/π=0.363 (check: 0.363+0.186+0.450≅1).

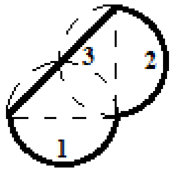


Fig. E2. Sketch to deduce the view factor F_{12} between two long hemicylinders (1) and (2).

Now we generalise this algebraic method of computing view factors in two-dimensional geometries to non-contact surfaces.

The crossed string method

For any two non-touching infinitely-long bands, 1 and 2 (Fig. 4), one can also find all the view factors from simple algebraic relations as in the triangular enclosure before, extending the result (8) to what is known as crossed-string method:

$$F_{12}=\frac{L_4+L_5-L_3-L_6}{2L_1}=\frac{\sum\text{crossed strings}-\sum\text{uncrossed strings}}{2\times\left(\text{source string}\right)} \qquad (9)$$

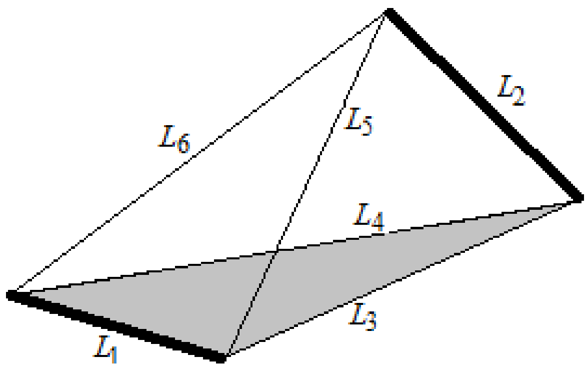


Fig. 4. Sketch used to deduce F_{12} in the general case of two infinitely long bands.

The result (9) is deduced by applying the triangular relation (8) to triangle 1–3–4 (shadowed in Fig. 4) and triangle 1–5–6, plus the closure relation to the quadrilateral 1–3–2–6 (F_{13}+F_{12}+F_{16}=1), namely:

$$\left.\begin{aligned}F_{13}&=\frac{L_1+L_3-L_4}{2L_1}\\F_{16}&=\frac{L_1+L_6-L_5}{2L_1}\end{aligned}\right\}\rightarrow\quad F_{12}=1-F_{12}-F_{12}=\frac{L_4+L_5-L_3-L_6}{2L_1} \qquad (10)$$

This procedure to compute view factors in two-dimensional configurations (the crossed-string method), was first developed by H.C. Hottel in the 1950s. The extension to non-planar surfaces 1 and 2 is as already presented for triangular enclosures. A further extension is possible to cases where there are obstacles (two-dimensional, of course) partially protruding into sides 3 and/or 6 in the quadrilateral 1–3–2–6 (Fig. 4); it suffices to account for the real curvilinear length of each string when stretched over the obstacles, as shown in the following example.

Example 3. Find the view factor between two long parallel cylinders of equal radii R, separated a distance $2\sqrt{2}\,R$ between centres, using the crossed-string method.

Sol. With this clever separation, angle θ in Fig. E1 happens to be $\theta=\pi/4$ (45º), making calculations simpler. We get F_{12} from (10) by substituting $L_1=2\pi R$ (the source cylinder), L_4 and L_5 (the crossing strings) each by the length $abcde$, and L_3 and L_6 (the non-crossing strings) each by $2\sqrt{2}\,R$ between. The length $abcde$ is composed of arc ab, segment bc, and so on, which in our special case is $ab=R\theta=(\pi/4)R$ $bc=R$, and $abcde=2(ab+cd)=(\pi/2)R+2R$, and finally $F_{12}=(L_4+L_5-L_3-L_6)/(2L_1)=(2abcde-4\sqrt{2}\,R)/(4\pi R)=(\pi R+4R-4\sqrt{2}\,R)/(4\pi R)=1/4+(1-\sqrt{2})/\pi=0.12$, as can be checked with the general expression for cylinders in the compilation following.

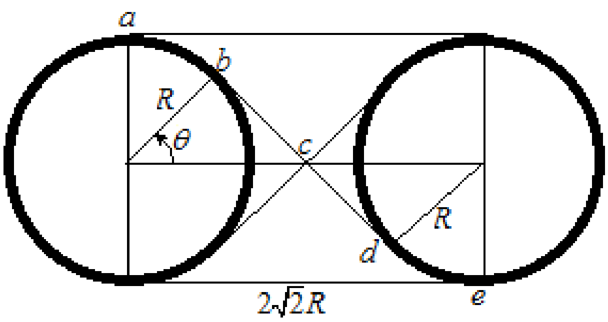


Fig. E1. Sketch used to deduce F_{12} between two infinitely long parallel cylinders.

View factor with an infinitesimal surface: the unit-sphere and the hemicube methods

The view factor from an infinitesimal surface dA_1 to a finite surface A_2, $F_{12}=\int\cos(\beta_1)d\Omega_{12}/\pi$, shows that it is the same for any surface subtending a slid angle Ω_{12} in the direction β_1. Hence, a convenient way to compute F_{12}, first proposed by Nusselt in 1928, is to find the conical projection of A_2 on a sphere of arbitrary radius r centred on dA_1 (this projection has the same Ω_{12}), and then project this spherical patch on the base plane containing dA_1, as sketched in Fig. 5a (the conical projection on the sphere is A_s, and its normal projection on the base plane is A_p). The view factor F_{12} is therefore the fraction of the circle occupied by A_p, i.e. $F_{12}=A_p/(\pi r^2)$. This was originally an experimental method: if an opaque reflective hemisphere is put on top of dA_1, and a photography is taken from the zenith, the measure of the reflected image of A_2 (A_p), divided by the area of the circle, yields the view factor (in practice, the radius of the sphere was taken as unity, so the unit-sphere naming).

The same reasoning can be followed changing the intermediate hemisphere by any other convex surface covering the 2π steradians over dA_1, e.g. the hemicube method uses an intermediate hexahedron or cube centred in dA_1, what can simplify computations, since the discretization of the planar faces in small patches is simpler, and the view factors of every elementary patch in a rectangular grid on the top and lateral faces of the hemicube can be precomputed; Fig. 5b shows the hemicube geometry in comparison with the unit-sphere geometry.

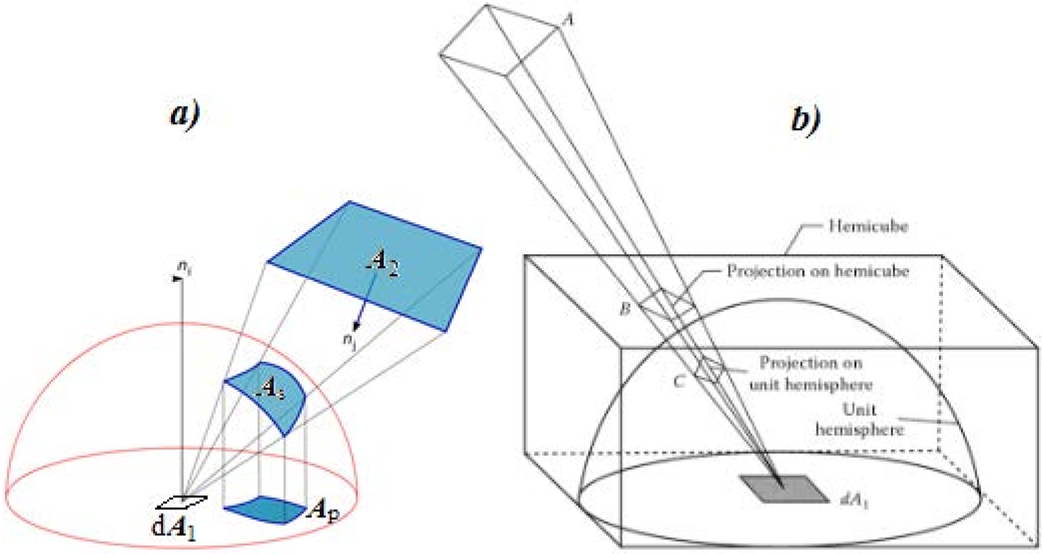
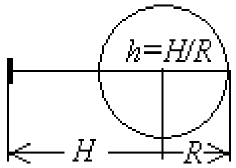
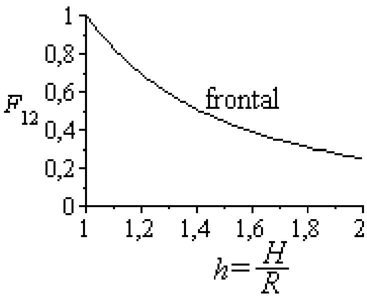


Fig. 5. a) The unit-sphere method, and b) its comparison with the hemicube method (Howell et al.).

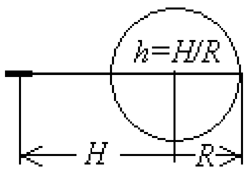
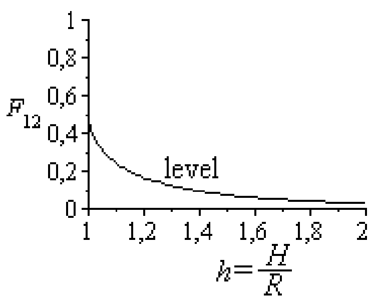
WITH SPHERES

Patch to a sphere

Frontal

Case	View factor	Plot
From a small planar surface facing a sphere of radius R, at a distance H from centres, with $h \equiv H/R$. 	$F_{12}=\frac{1}{h^2}$ (e.g. for h=2, F_{12}=1/4)	

Level

Case	View factor	Plot
From a small planar plate (one face or both) level to a sphere of radius R, at a distance H from centres, with $h \equiv H/R$. 	$F_{12}=\frac{1}{\pi}\left(\arctan\frac{1}{x}-\frac{x}{h^2}\right)$ with $x\equiv\sqrt{h^2-1}$ ($F_{12}\big\|_{h\to 1}\to\frac{1}{2}-\frac{2\sqrt{2}}{\pi}\sqrt{h-1}$) (e.g. for h=2, F_{12}=0.029)	

Tilted

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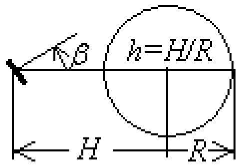
From a small planar surface tilted to a sphere of radius *R*, at a distance *H* from centres, with $h \equiv H/R$; the tilting angle β is between the normal and the line of centres.



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For the facing surface:
-if $|\beta| \le \arccos(1/h)$ (i.e. plane not cutting the sphere),

$$F_{12} = \frac{\cos\beta}{h^2}$$

-if $|\beta| > \arccos(1/h)$ (i.e. plane cutting the sphere),

$$F_{12} = \frac{1}{\pi h^2}\left(\cos\beta \arccos y - x\sin\beta\sqrt{1-y^2}\right) + \frac{1}{\pi}\arctan\left(\frac{\sin\beta\sqrt{1-y^2}}{x}\right)$$

with $x \equiv \sqrt{h^2-1}, \quad y \equiv -x\cot(\beta)$

For the whole plate (the two surfaces):
-if $|\beta| \le \arccos(1/h)$ (i.e. plane not cutting the sphere, hence $F_{12}=0$ for the back side),

$$F_{12} = \frac{\cos\beta}{2h^2}$$

-if $|\beta| > \arccos(1/h)$ (i.e. plane cutting the sphere), F_{12} can be obtained as the semisum of values for the facing surface and for the opposite surface, the latter obtained as the complement angle (i.e. with $\beta \to \pi - \beta$).

(e.g. for h=1.5 and $\beta=\pi/3$ (60º):
F_{12}=0.226 for the facing side,
F_{12}=0.004 for the opposite side, and
F_{12}=0.115 for the two-side plate)

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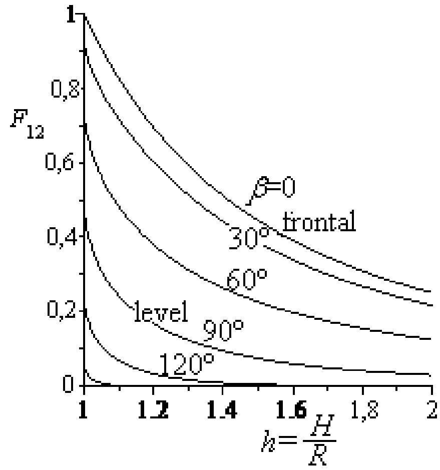

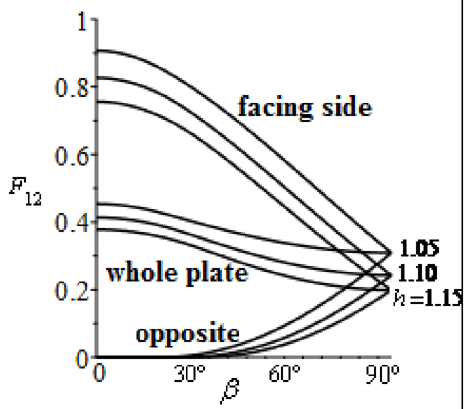

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Patch to a spherical cap

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From a small planar plate facing a spherical cap subtending a half-cone angle α (or any other surface subtending the same solid angle).

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$$F_{12} = \sin^2\alpha$$

(e.g. for α=45º, F_{12}=1/2)

Notice that the case 'patch to frontal sphere' above, can be recovered in our case with α_{max}=arcsin(R/H).

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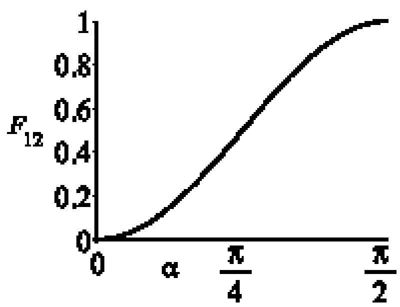

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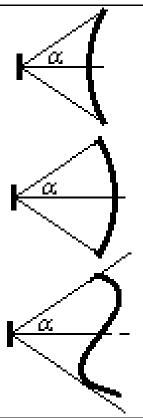
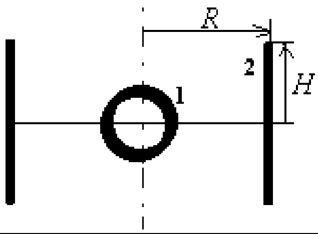
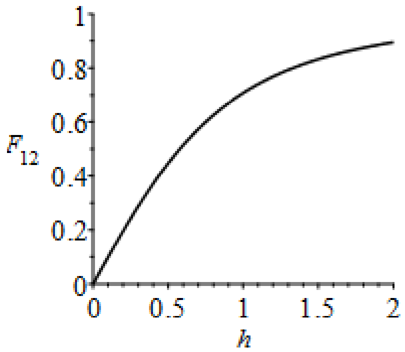
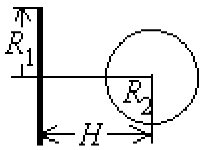
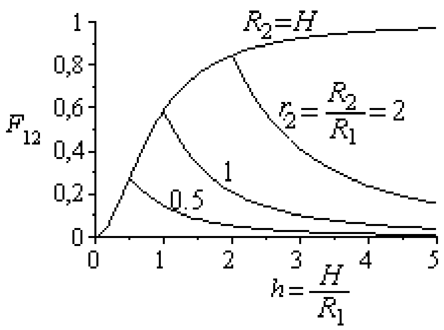
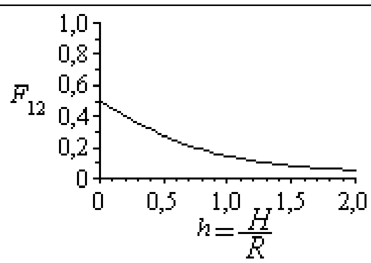
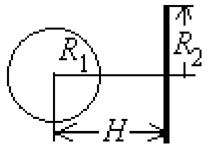
		

Sphere to concentric external cylinder

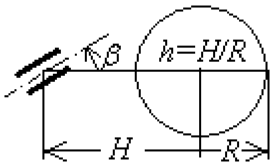
Case	View factor	Plot
From a sphere (1) to interior surface of a concentric cylinder (2) of radius R and height $2H$, $h \equiv H/R$. Sphere radius, R_{sph}, is irrelevant but must be $R_{sph} \leq H$. 	$F_{12} = \frac{h}{\sqrt{1+h^2}}$ (e.g. for H=R, i.e. h =1, F_{12}=0.707)	

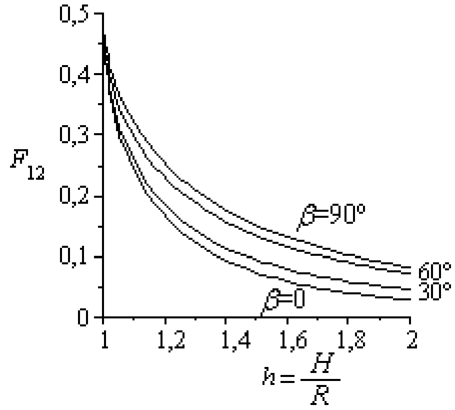
Disc to frontal sphere

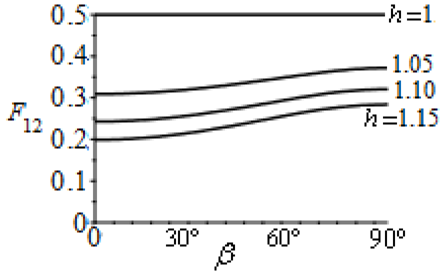
Case	View factor	Plot
From a disc of radius R_1 to a frontal sphere of radius R_2 at a distance H between centres (it must be $H>R_1$), with $h \equiv H/R_1$ and $r_2 \equiv R_2/R_1$. 	$F_{12} = 2r_2^2\left(1-\frac{1}{\sqrt{1+\frac{1}{h^2}}}\right)$ (e.g. for h=r_2=1, F_{12}=0.586)	
From a sphere of radius R_1 to a frontal disc of radius R_2 at a distance H between centres (it must be $H>R_1$, but does not depend on R_1), with $h \equiv H/R_2$.	$F_{12} = \frac{1}{2}\left(1-\frac{1}{\sqrt{1+\frac{1}{h^2}}}\right)$ (e.g. for R_2=H and $R_1 \leq H$, F_{12}=0.146)	

<table>
<tr><td>
</td><td></td><td></td></tr>
</table>

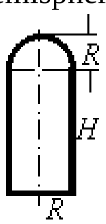
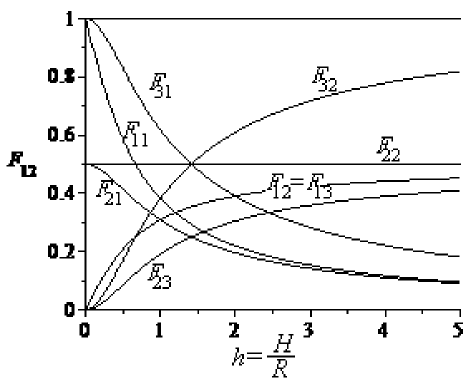
Cylinder to large sphere

<table>
<tr><th>Case</th><th>View factor</th><th>Plot</th></tr>
<tr><td>From a small cylinder (external lateral area only), at an altitude H=hR and tilted an angle β, to a large sphere of radius R, β is between the cylinder axis and the line of centres).

</td><td>Coaxial (β=0):
$$F_{12}=\frac{1}{2}-\frac{s}{\pi h}-\frac{\arcsin(s)}{\pi}$$ with $s\equiv\sqrt{1-\frac{1}{h^2}}$
Perpendicular ($\beta=\pi/2$):
$$F_{12}=\frac{4}{\pi^2}\int_0^{\frac{1}{h}}\frac{x\mathrm{E}(x)\,\mathrm{d}x}{\sqrt{1-x^2}}$$ with elliptic integrals E(x).

Tilted cylinder:
$$F_{12}=\int_{\theta=0}^{\arcsin\left(\frac{1}{h}\right)}\int_{\phi=0}^{2\pi}\frac{\sin(\theta)\sqrt{1-z^2}\,\mathrm{d}\theta\mathrm{d}\phi}{\pi^2}$$ with
$$z\equiv\cos(\theta)\cos(\beta)+ +\sin(\theta)\sin(\beta)\cos(\phi)$$ (e.g. for h=1 and any β, F_{12}=1/2)</td><td>

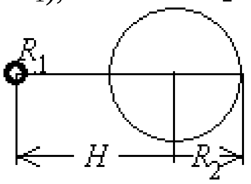
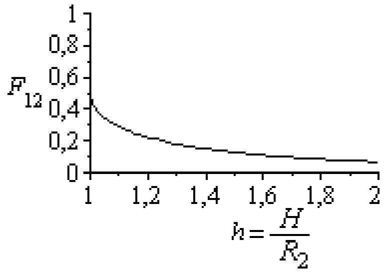

</td></tr>
</table>

Cylinder to its hemispherical closing cap

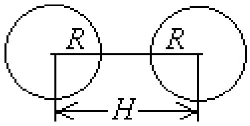
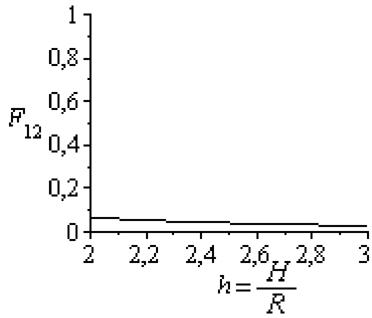
<table>
<tr><th>Case</th><th>View factor</th><th>Plot</th></tr>
<tr><td>From a finite cylinder (surface 1) of radius R and height H, to its hemispherical closing cap (surface 2), with r=R/H. Let surface 3 be the base, and surface 4 the virtual base of the hemisphere.

</td><td>$$F_{11}=1-\frac{\rho}{2},\ F_{12}=F_{13}=F_{14}=\frac{\rho}{4}$$ $$F_{21}=\frac{\rho}{4r},\ F_{22}=\frac{1}{2},\ F_{23}=\frac{1}{2}-\frac{\rho}{4r},$$ $$F_{31}=\frac{\rho}{2r},\ F_{32}=1-\frac{\rho}{2r},\ F_{34}=1-\frac{\rho}{2r}$$ $$\text{with }\rho=\frac{\sqrt{4r^2+1}-1}{r}$$ (e.g. for R=H, F_{11}=0.38, F_{12}=0.31, F_{21}=0.31, F_{22}=0.50, F_{23}=0.19, F_{31}=0.62, F_{32}=0.38, F_{34}=0.38)</td><td>
</td></tr>
</table>

Sphere to sphere

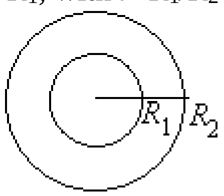
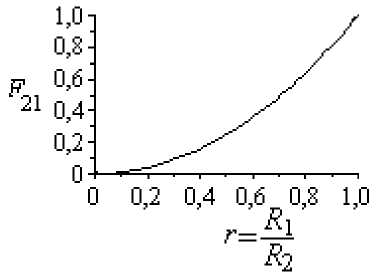
Small to very large

Case	View factor	Plot
From a small sphere of radius R_1 to a much larger sphere of radius R_2 at a distance H between centres (it must be $H>R_2$, but does not depend on R_1), with $h \equiv H/R_2$. 	$F_{12} = \frac{1}{2}\left(1-\sqrt{1-\frac{1}{h^2}}\right)$ (e.g. for $H=R_2$, $F_{12}=1/2$)	

Equal spheres

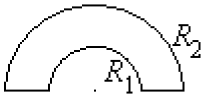
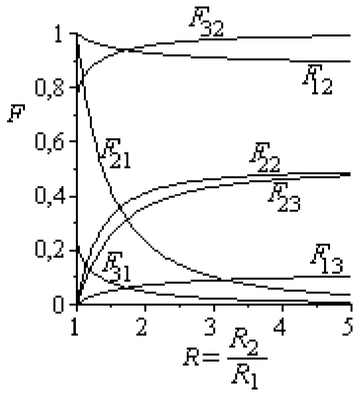
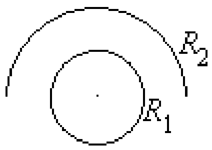
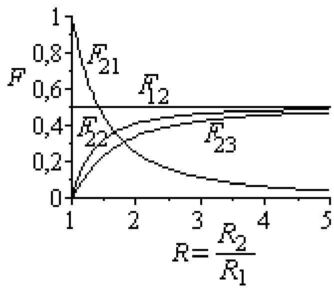
Case	View factor	Plot
From a sphere of radius R to an equal sphere at a distance H between centres (it must be $H>2R$), with $h \equiv H/R$. 	$F_{12} = \frac{1}{2}\left(1-\sqrt{1-\frac{1}{h^2}}\right)$ (e.g. for $H=2R$, $F_{12}=0.067$)	

Concentric spheres

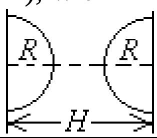
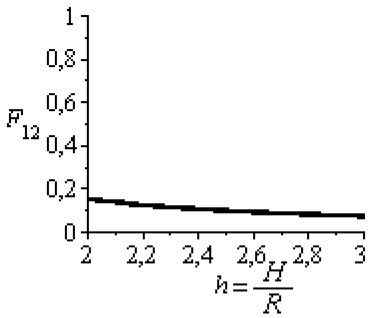
Case	View factor	Plot
Between concentric spheres of radii R_1 and $R_2>R_1$, with $r \equiv R_1/R_2<1$. 	$F_{12}=1$ $F_{21}=r^2$ $F_{22}=1-r^2$ (e.g. for $r=1/2$, $F_{12}=1$, $F_{21}=1/4$, $F_{22}=3/4$)	

Hemispheres

Case	View factor	Plot

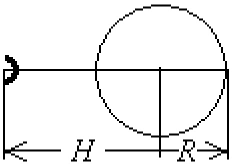
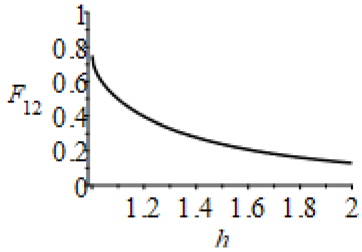
From a hemisphere of radius R_1 to a larger concentric hemisphere of radius $R_2>R_1$, with $R\equiv R_2/R_1>1$. Let the closing planar annulus be surface 3. 	$F_{12}=1-\frac{\rho}{4}$, $F_{13}=\frac{\rho}{4}$, $F_{21}=\frac{1}{R^2}\left(1-\frac{\rho}{4}\right)$, $F_{22}=\frac{1}{2}\left(1-\frac{1-\rho}{R^2}\right)$, $F_{23}=\frac{1}{2}\left(1-\frac{1}{R^2}\right)\left(1-\frac{\rho}{2\left(R^2-1\right)}\right)$, $F_{31}=\frac{\rho}{2R^2}$, $F_{32}=1-\frac{\rho}{2\left(R^2-1\right)}$ with $\rho=\frac{1}{2}-\frac{1}{\pi}\left[\sqrt{R^2-1}-\left(R^2-2\right)\arcsin\left(\frac{1}{R}\right)\right]$ Limit for $R\to\infty$: F_{12}=7/8=0.875, F_{13}=1/8 (e.g. for R=2, F_{12}=0.93, F_{21}=0.23, F_{13}=0.07, F_{31}=0.05, F_{32}=0.95, F_{23}=0.36,F_{22}=0.41)	
From a sphere of radius R_1 to a larger concentric hemisphere of radius $R_2>R_1$, with $R\equiv R_2/R_1>1$. Let the enclosure be '3'. 	F_{12}=1/2, F_{13}=1/2, $F_{21}=1/R^2$, $F_{23}=1-F_{21}-F_{22}$, $F_{22}=\frac{1}{2}\left(1-\frac{1-\rho}{R^2}\right)$ with $\rho=\frac{1}{2}-\frac{1}{\pi}\left[\sqrt{R^2-1}-\left(R^2-2\right)\arcsin\left(\frac{1}{R}\right)\right]$ (e.g. for R=2, F_{12}=1/2, F_{21}=1/4, F_{13}=1/2, F_{23}=0.34,F_{22}=0.41)	

Equal frontal hemispheres

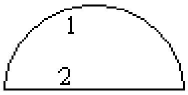
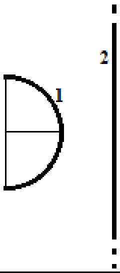
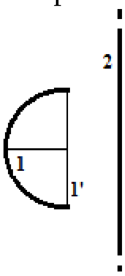
Case	View factor	Plot
From a hemisphere of radius R to an equal frontal hemisphere at a distance H between centres (it must be $H>2R$), with $h\equiv H/R$. 	$F_{12}=1-\sqrt{1-\frac{1}{h^2}}$ (e.g. for $H=2R$, F_{12}=0.134)	

Small hemisphere frontal to large sphere

Case	View factor	Plot

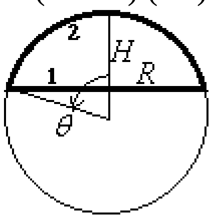
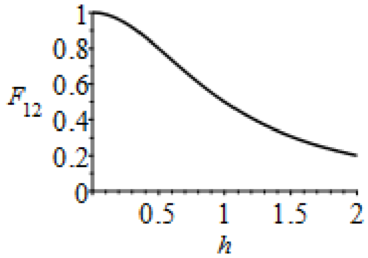
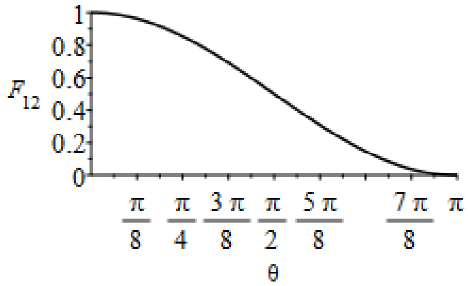
From a small hemisphere (one face) to a large frontal sphere of radius *R* at a distance $H \geq R$ between centres, with $h \equiv H/R$. 	$F_{12} = \frac{1}{2}\left(1 - \sqrt{1 - \frac{1}{h^2}} + \frac{1}{2h^2}\right)$ (e.g. for $H=R$, $F_{12}=3/4$)	

Hemisphere to plane

Case	View factor	Plot
From a hemisphere of radius *R* (surface 1) to its base circle (surface 2). 	$F_{21}=1$ $F_{12}=A_2F_{21}/A_1=1/2$ $F_{11}=1-F_{12}=1/2$	
From a convex hemisphere (1) to an infinite plane (2). Let the enclosure be ‘3’. 	(coincides with Small hemisphere frontal to large sphere at $H=R$) $F_{12}=3/4=0.75$ $F_{13}=1/4=0.25$ $F_{21}=0$, $F_{23}=1$	
From a concave hemisphere (1) to an infinite plane (2). 	$F_{12}=F_{11'}=1/2$ $F_{11}=1/2$	

Spherical cap to base disc

Case	View factor	Plot

From a disc of radius R (surface 1) to a closing spherical cap of height H (surface 2), with $h=H/R$. The radius of the sphere is $(R^2+H^2)/(2H)$. 	$F_{12}=1$ $F_{21}=\frac{A_1}{A_2}F_{12}=\frac{R^2}{R^2+H^2}=\frac{1}{1+h^2}=\cos^2\frac{\theta}{2}$ $F_{22}=\frac{h^2}{1+h^2}=\sin^2\frac{\theta}{2}$ (e.g. for h=1, i.e. θ=90º, $F_{21}=F_{22}$=1/2)	

WITH CYLINDERS

Cylinder to large sphere

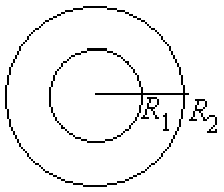
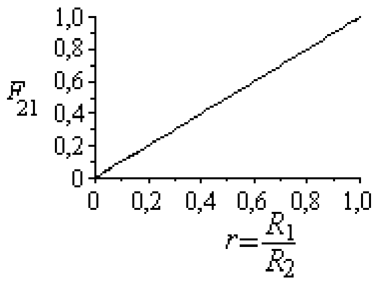
See results under ‘Cases with spheres’.

Cylinder to its hemispherical closing cap

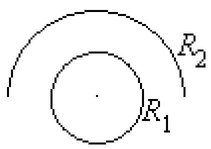
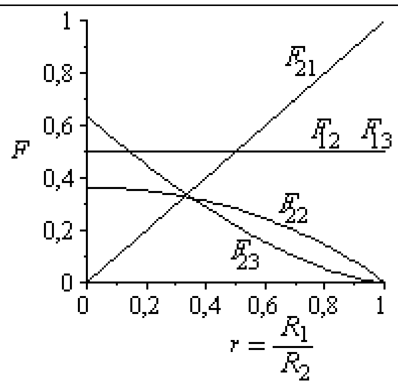
See results under ‘Cases with spheres’.

Very-long cylinders

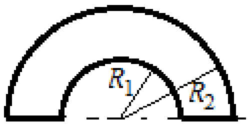
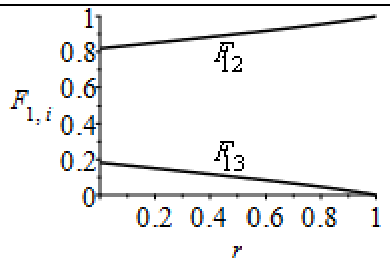
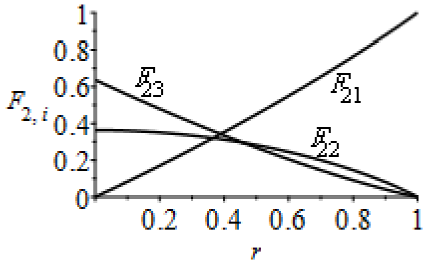
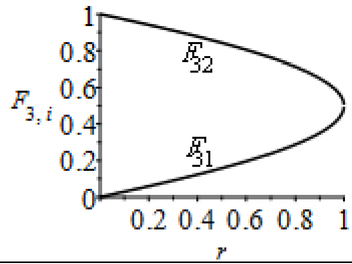
Concentric cylinders

Case	View factor	Plot
Between concentric infinite cylinders of radii R_1 and $R_2>R_1$, with $r\equiv R_1/R_2<1$. 	$F_{12}=1$ $F_{21}=r$ $F_{22}=1-r$ (e.g. for r=1/2, F_{12}=1, F_{21}=1/2, F_{22}=1/4)	

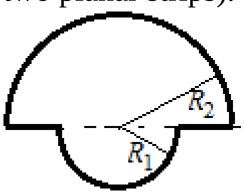
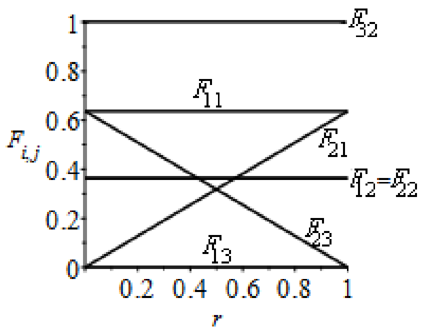
Concentric cylinder to hemi-cylinder

Case	View factor	Plot
From very-long cylinder of radius R_1 to concentric hemi-cylinder of radius $R_2>R_1$, with $r\equiv R_1/R_2<1$. Let the enclosure be ‘3’. 	F_{12}=1/2, $F_{21}=r$, F_{13}=1/2, $F_{23}=1-F_{21}-F_{22}$, $F_{22}=1-\frac{2}{\pi}\left(\sqrt{1-r^2}+r\arcsin r\right)$ (e.g. for r=1/2, F_{12}=1/2, F_{21}=1/2, F_{13}=1/2, F_{23}=0.22, F_{22}=0.28)	

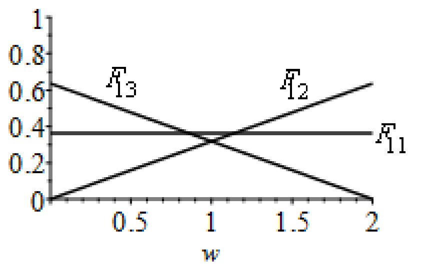
Concentric frontal hemi-cylinders

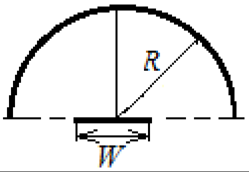
Case	View factor	Plot
From very-long hemi-cylinder of radius R_1 to concentric hemi-cylinder of radius $R_2>R_1$, with $r\equiv R_1/R_2<1$. Let '3' be the closing surface (i.e. the two planar strips).  	$F_{12}=1-\frac{\arccos(r)}{\pi}+\frac{1}{\pi r}\left(\sqrt{1-r^2}+r-1\right)$ (limit for $r\to 0$: $F_{12}=\frac{1}{2}+\frac{1}{\pi}=0.82$), $F_{11}=0$, $F_{13}=1-F_{12}$, $F_{31}=\frac{\pi r F_{13}}{2(1-r)}$, $F_{33}=0$, $F_{32}=1-F_{31}$, $F_{23}=\frac{2(1-r)F_{32}}{\pi}$, $F_{21}=rF_{12}$, $F_{22}=1-F_{21}-F_{23}$, $F_{22}=1-r+\frac{2}{\pi}\left(r\arccos(r)-\sqrt{1-r^2}\right)$ (e.g. for $r=1/2$: $F_{11}=0$, $F_{12}=0.90$, $F_{13}=0.10$, $F_{21}=0.45$, $F_{22}=0.28$, $F_{23}=0.27$ $F_{31}=0.16$, $F_{32}=0.84$, $F_{33}=0$)	  

Concentric opposing hemi-cylinders

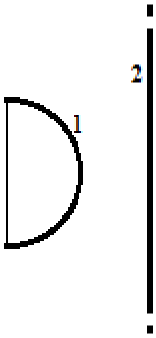
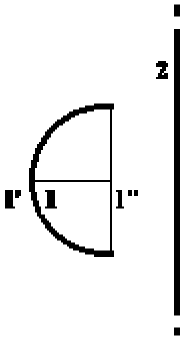
Case	View factor	Plot
From very-long hemi-cylinder of radius R_1 to concentric hemi-cylinder of radius $R_2>R_1$, with $r\equiv R_1/R_2<1$. Let '3' be the closing surface (i.e. the two planar strips).  	$F_{11}=\frac{2}{\pi}=0.64$, $F_{12}=1-\frac{2}{\pi}=0.36$, $F_{13}=0$, $F_{21}=\frac{2r}{\pi}$, $F_{22}=1-\frac{2}{\pi}=0.36$, $F_{23}=\frac{2(1-r)}{\pi}$ $F_{31}=0$, $F_{32}=1$, $F_{33}=0$ (e.g. for $r=1/2$: $F_{11}=0.64$, $F_{12}=0.36$, $F_{13}=0$, $F_{21}=0.32$, $F_{22}=0.36$, $F_{23}=0.32$ $F_{31}=0$, $F_{32}=1$, $F_{33}=0$)	

Hemi-cylinder to central strip

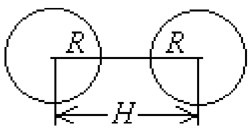
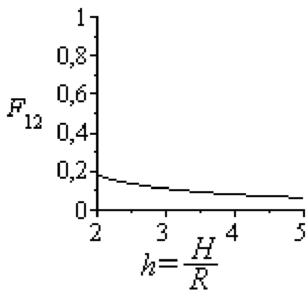
Case	View factor	Plot
From very-long hemi-cylinder of radius R to symmetrically placed central strip of width W, with $w\equiv W/R<2$. Let the enclosure be '3'.	$F_{11}=1-\frac{2}{\pi}=0.363$, $F_{12}=\frac{w}{\pi}$, $F_{13}=\frac{2-w}{\pi}$) (e.g. for $w=1$, $F_{12}=F_{13}=0.318$, $F_{11}=0.363$)	

Hemi-cylinder to infinite plane

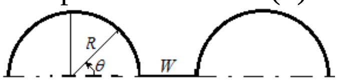
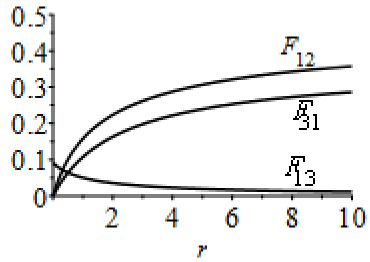
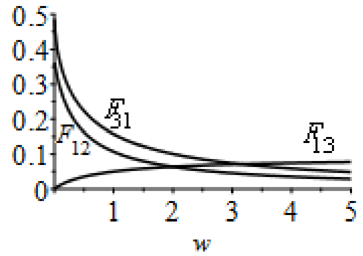
Case	View factor	Plot
From convex hemi-cylinder to frontal plane. Let the enclosure be '3'. 	$F_{11}=0$ $F_{12}=\frac{1}{2}+\frac{1}{\pi}=0.82$ $F_{13}=\frac{1}{2}-\frac{1}{\pi}=0.18$	
From hemi-cylinder to plane. Let 1 be the concave side, 1' the convex side, 1" the diametrical section, and 3 the enclosure. 	Concave side: $F_{11}=1-\frac{2}{\pi}=0.36$ $F_{12}=F_{11''}=\frac{2}{\pi}=0.64$ $F_{13}=0$ Convex side: $F_{1'2}=\frac{1}{2}-\frac{1}{\pi}=0.18$ $F_{1'3}=\frac{1}{2}+\frac{1}{\pi}=0.82$	

Equal external cylinders

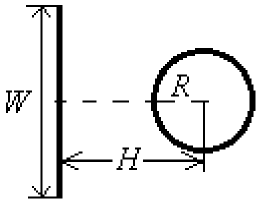
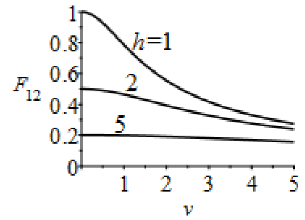
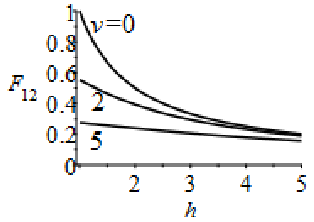
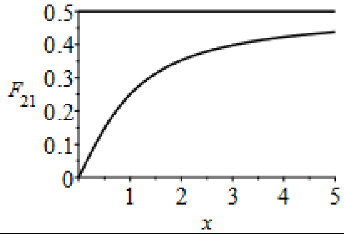
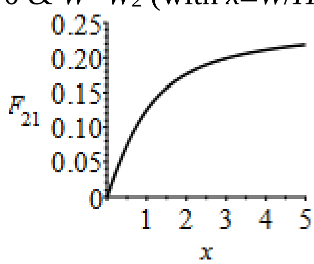
Case	View factor	Plot
From a cylinder of radius *R* to an equal cylinder at a distance *H* between centres (it must be *H*>2*R*), with *h*≡*H*/*R*.  <u>Note</u>. See the crossing-	$F_{12}=\frac{\sqrt{h^2-4}-h+2\arcsin\frac{2}{h}}{2\pi}$ (e.g. for $H=2R$, $F_{12}=1/2-1/\pi=0.18$)	

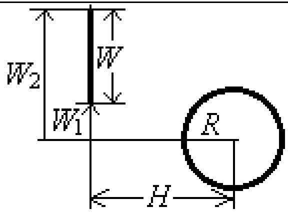
string method, above.		

Equal external hemi-cylinders

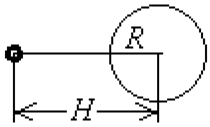
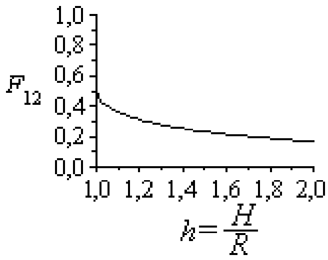
Case	View factor	Plot
From a hemi-cylinder of radius R to an equal hemi-cylinder separated a distance W, with $r\equiv R/W$ ($w\equiv W/R=1/r$). Let the strip in between be (3).  	$F_{12}=1-\frac{2}{\pi}+\frac{4}{\pi r}\left(1-\sqrt{1+r}+\frac{r}{2}\arccos\frac{r}{2+r}\right)$ (limit for $r\to\infty$: $F_{12}=1-\frac{2}{\pi}=0.36$) $F_{13}=\frac{1}{2\pi r}\left(1+r\arccos\frac{r}{1+r}-\sqrt{1+2r}\right)$ (limit for $r\to 0$: $F_{13}=\frac{1}{4}-\frac{1}{2\pi}=0.091$) (e.g. for $r=1$, $F_{12}=0.11$, $F_{13}=0.05$)	 

Planar strip to cylinder

Case	View factor	Plot
From frontal surface of strip (1) of width W to a cylinder (2) of radius R at a distance H between centres, with $v\equiv W/(2R)$ and $h\equiv H/R>1$.  	$F_{12}=\frac{\arctan\left(\frac{v}{h}\right)}{v}$ $F_{21}=\frac{\arctan\left(\frac{v}{h}\right)}{\pi}=\frac{\arctan(x)}{\pi}$ with $x\equiv v/h=(W/2)/H$ (e.g. for $W=R=H$, i.e. $v=1/2$, $h=1$, $x=1/2$, $F_{12}=0.927$, $F_{21}=0.148$)	  
From frontal surface of off-centre strip of width W to a cylinder of radius R at a distance H to the strip plane. The solution is the composition of a strip of with W_1 and a strip of width W_2, with $v_1\equiv W_1/R$, $v_2\equiv W_2/R$, $h\equiv H/R>1$, $x_1\equiv W_1/H$, $x_2\equiv W_2/H$.	For $W=W_2-W_1$: $F_{12}=\frac{\arctan(x_2)-\arctan(x_1)}{v_2-v_1}$ $F_{21}=\frac{\arctan\left(\frac{v_2}{h}\right)-\arctan\left(\frac{v_1}{h}\right)}{2\pi}$ For $W=W_2+W_1$: $F_{12}=\frac{\arctan(x_2)+\arctan(x_1)}{v_2+v_1}$	For $W_1=0$ & $W=W_2$ (with $x\equiv W/H$): 

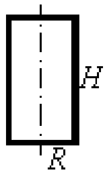
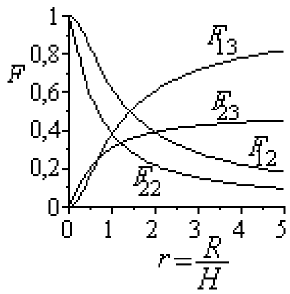
	$$F_{21}=\frac{\arctan\left(\frac{v_2}{h}\right)-\arctan\left(\frac{v_1}{h}\right)}{2\pi}$$ (e.g. for W_1=0 & $W_2=R=H$, i.e. v_1=0, v_2=1, h=1, x_1=0, x_2=1, F_{12}=0.463, F_{21}=0.074)	

Wire to parallel cylinder

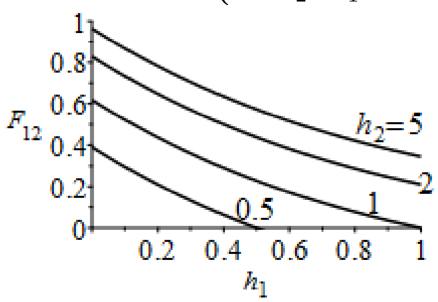
Case	View factor	Plot
From a small infinite long cylinder to an infinite long parallel cylinder of radius R, with a distance H between axes, with $h\equiv H/R$. 	$$F_{12}=\frac{\arcsin\frac{1}{h}}{\pi}$$ (e.g. for $H=R$, F_{12}=1/2)	

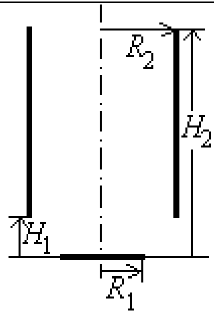
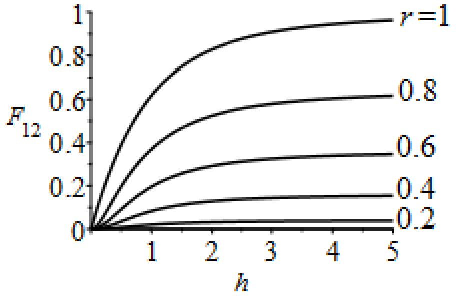
Finite cylinders

Base to lateral surface

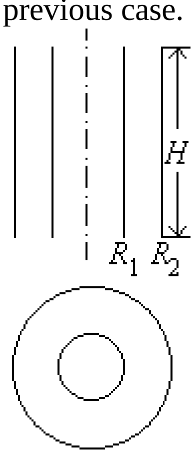
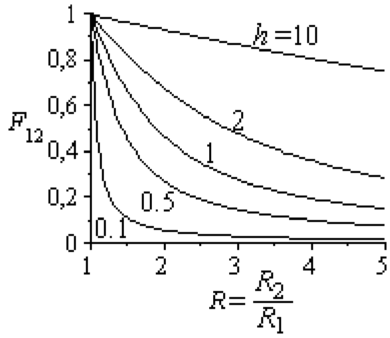
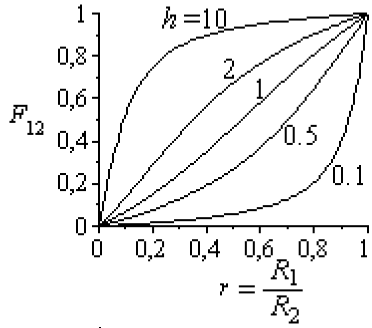
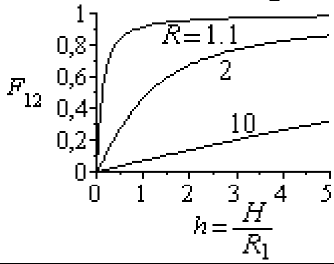
Case	View factor	Plot
From base (1) to lateral surface (2) in a cylinder of radius R and height H, with $r\equiv R/H$. Let (3) be the opposite base. 	$$F_{12}=\frac{\rho}{2r},\ F_{13}=1-\frac{\rho}{2r},$$ $$F_{21}=\frac{\rho}{4},\ F_{22}=1-\frac{\rho}{2},\ F_{23}=\frac{\rho}{4}$$ with $\rho=\frac{\sqrt{4r^2+1}-1}{r}$ (e.g. for $R=H$, F_{12}=0.62, F_{21}=0.31, F_{13}=0.38, F_{22}=0.38)	

Disc to coaxial cylinder

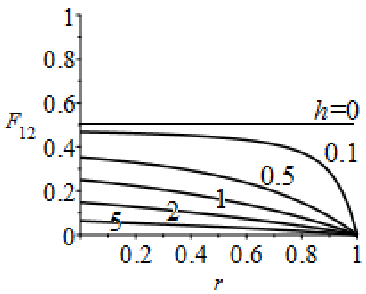
Case	View factor	Plot
From disc (1) of radius R_1 at a distance H_1, to internal lateral surface (2) of a coaxial cylinder of radius $R_2>R_1$ and height H_2-H_1, with $h_1\equiv H_1/R_1$, $h_2\equiv H_2/R_1$, $r\equiv R_2/R_1$.	$$F_{12}=\frac{1}{2}\left[(x_2-x_1)-(y_2-y_1)\right]$$ with $$x_1=1+r^2+h_1^2,\quad y_1=\sqrt{x_1^2-4r^2}$$ $$x_2=1+r^2+h_2^2,\quad y_2=\sqrt{x_2^2-4r^2}$$ (e.g. for H_1=0 & $H_2=R_2=R_1$, i.e. for r=1, h_1=0 & h_2=1, F_{12}=0.62)	Disc of same radius (i.e. $R_2=R_1$ and r=1)  Disc at the base (i.e. H_1=0, with $h=h_2$)

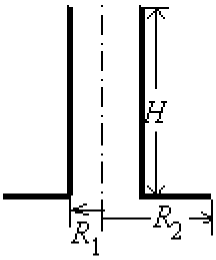
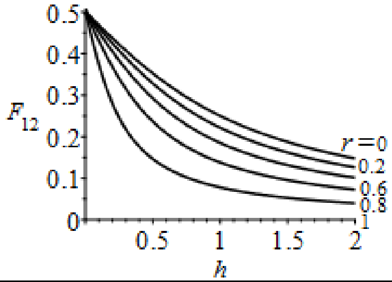
		

Equal finite concentric cylinders

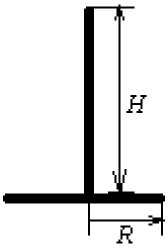
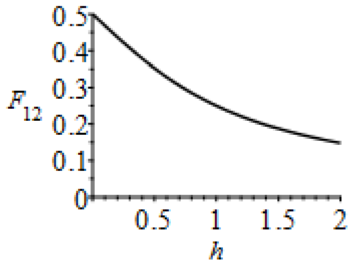
Case	View factor	Plot
Between finite concentric cylinders of radius R_1 and R_2>R_1 and height H, with h=H/R_1 and R=R_2/R_1. Let the enclosure be '3'. For the inside of '1', see previous case. 	$F_{12}=1-\frac{1}{\pi}\left(\arccos\frac{f_2}{f_1}-\frac{f_4}{2h}\right)$, $F_{13}=1-F_{12}$, $F_{22}=1-\frac{1}{R}+\frac{2}{\pi R}\arctan\frac{2\sqrt{R^2-1}}{h}-\frac{hf_7}{2\pi R}$, $F_{23}=1-F_{21}-F_{22}$ with $f_1=h^2+R^2-1$, $f_2=h^2-R^2+1$, $f_3=\sqrt{(f_1+2)^2-4R^2}$, $f_4=f_3\arccos\frac{f_2}{Rf_1}+f_2\arcsin\frac{1}{R}-\frac{\pi f_1}{2}$, $f_5=\sqrt{\frac{4R^2}{h^2}+1}$, $f_6=1-\frac{2h^2}{R^2\left(h^2+4R^2-4\right)}$, $f_7=f_5\arcsin f_6-\arcsin\left(1-\frac{1}{R^2}\right)+\frac{\pi}{2}(f_5-1)$ (e.g. for R_2=2R_1 and H=2R_1, F_{12}=0.64, F_{21}=0.34, F_{13}=0.33, F_{23}=0.43, F_{22}=0.23)	

Outer surface of cylinder to annular disc joining the base

Case	View factor	Plot
From external lateral surface (1) of a cylinder of radius R_1 and height H, to annular disc of radius R_2, with r≡R_1/R_2, h≡H/R_2.	$F_{12}=\frac{y}{8rh}-\frac{\left(z+x\arcsin(r)\right)}{4\pi rh}+\frac{1}{2\pi}\arccos\left(\frac{x}{y}\right)$, with $x=h^2+r^2-1$, $y=h^2-r^2+1$, $z=\sqrt{(x+2)^2-4r^2}\arccos\left(\frac{xr}{y}\right)$	

	(e.g. for $R_2=2R_1=2H$, i.e. $r=h=1/2$, $F_{12}=0.268$, $F_{21}=0.178$)	

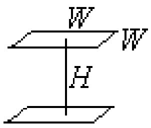
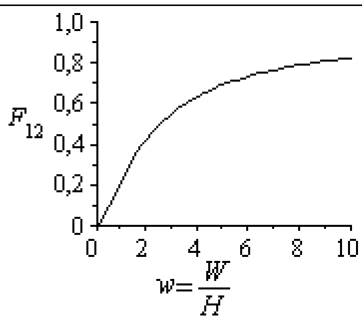
Cylindrical rod to coaxial disc at one end

Case	View factor	Plot
Thin rod (1) of height H, to concentric disc (2) of radius R placed at one end, with $h \equiv H/R$. 	$F_{12}=\frac{1}{4}-\frac{1}{2\pi}\arcsin\frac{h^2-1}{h^2+1}$, (e.g. for $R=H$, i.e. $h=1$, $F_{12}=1/4$)	

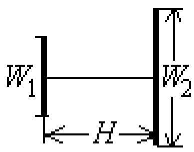
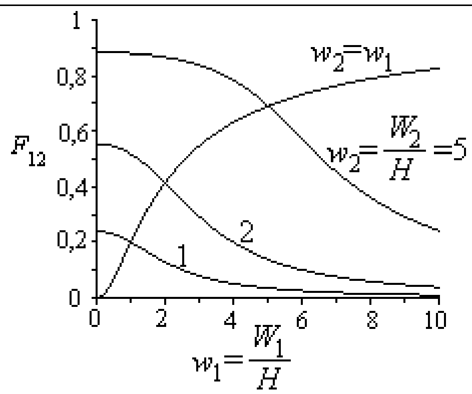
WITH PLATES AND DISCS

Parallel configurations

Equal square plates

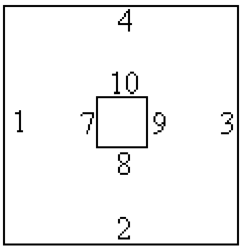
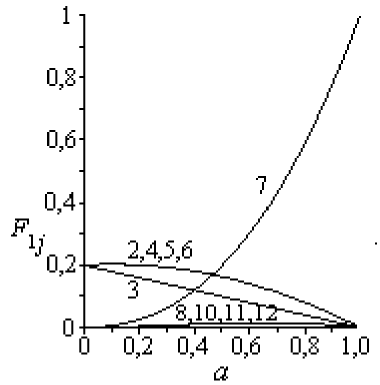
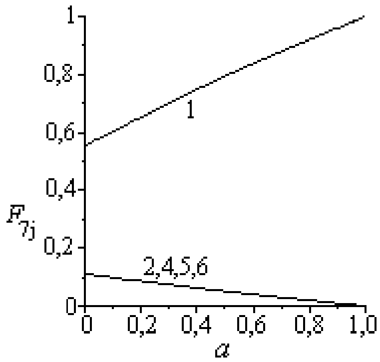
Case	View factor	Plot
Between two identical parallel square plates of side L and separation H, with $w=W/H$. 	$F_{12}=\frac{1}{\pi w^2}\left(\ln\frac{x^4}{1+2w^2}+4wy\right)$ with $x \equiv \sqrt{1+w^2}$ and $y \equiv x\arctan\frac{w}{x}-\arctan w$ (e.g. for $W=H$, $F_{12}=0.1998$)	

Unequal coaxial square plates

Case	View factor	Plot
From a square plate of side W_1 to a coaxial square plate of side W_2 at separation H, with $w_1=W_1/H$ and $w_2=W_2/H$. 	$F_{12}=\frac{1}{\pi w_1^2}\left(\ln\frac{p}{q}+s-t\right)$, with	

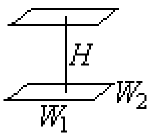
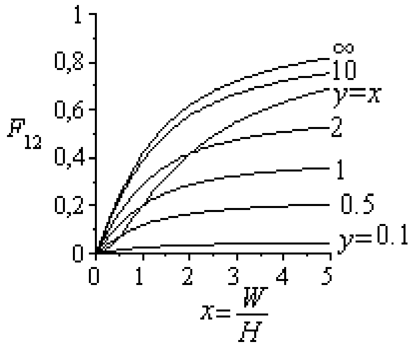
	$\begin{cases} p \equiv \left(w_1^2 + w_2^2 + 2\right)^2 \\ q \equiv \left(x^2+2\right)\left(y^2+2\right) \\ x \equiv w_2 - w_1, \quad y \equiv w_2 + w_1 \\ s \equiv u\left(x \arctan\frac{x}{u} - y\arctan\frac{y}{u}\right) \\ t \equiv v\left(x \arctan\frac{x}{v} - y\arctan\frac{y}{v}\right) \\ u \equiv \sqrt{x^2+4}, \quad v \equiv \sqrt{y^2+4} \end{cases}$ (e.g. for $W_1=W_2=H$, F_{12}=0.1998)	

Box inside concentric box

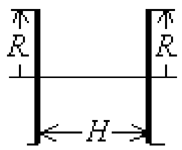
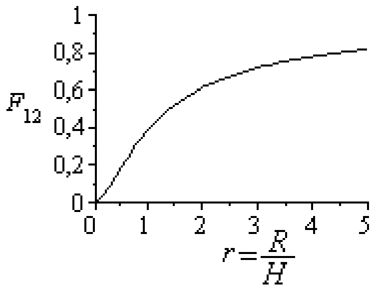
Case	View factor	Plot
Between all faces in the enclosure formed by the internal side of a cube box (faces 1-2-3-4-5-6), and the external side of a concentric cubic box (faces (7-8-9-10-11-12) of size ratio a≤1.  (A generic outer-box face #1, and its corresponding face #7 in the inner box, have been chosen.)	From an external-box face: $\begin{cases} F_{11}=0,\ F_{12}=x,\ F_{13}=y,\ F_{14}=x, \\ F_{15}=x,\ F_{16}=x,\ F_{17}=za^2,\ F_{18}=r, \\ F_{19}=0,\ F_{1,10}=r,\ F_{1,11}=r,\ F_{1,12}=r \end{cases}$ From an internal-box face: $\begin{cases} F_{71}=z,\ F_{72}=(1-z)/4,\ F_{73}=0,\ F_{74}=(1-z)/4, \\ F_{75}=(1-z)/4,\ F_{76}=(1-z)/4,\ F_{77}=0,\ F_{78}=0, \\ F_{79}=0,\ F_{7,10}=0,\ F_{7,11}=0,\ F_{7,12}=0 \end{cases}$ with z given by: $\begin{cases} z=F_{71}=\frac{(1-a)^2}{4\pi a^2}\left(\ln\frac{p}{q}+s+t\right) \\ p \equiv \left(2\frac{3-2a+3a^2}{(1-a)^2}\right)^2 \\ q \equiv 2\frac{18+12a+18a^2}{(1-a)^2} \\ s \equiv u\left(2\arctan\frac{2}{u} - w\arctan\frac{w}{u}\right) \\ t \equiv v\left(2\arctan\frac{2}{v} - w\arctan\frac{w}{v}\right) \\ u \equiv \sqrt{8},\ v \equiv \frac{\sqrt{8\left(1+a^2\right)}}{1-a},\ w \equiv 2\frac{1+a}{1-a} \end{cases}$ and:	From face 1 to the others:  From face 7 to the others: 

	$\begin{cases} r \equiv a^2(1-z)/4 \\ y \simeq 0.2(1-a) \\ x \equiv \left(1-y-za^2-4r\right)/4 \end{cases}$ (e.g. for a=0.5, F_{11}=0, F_{12}=0.16, F_{13}=0.10, F_{14}=0.16, F_{15}=0.16, F_{16}=0.16, F_{17}=0.20, F_{18}=0.01, F_{19}=0, $F_{1,10}$=0.01, $F_{1,11}$=0.01, $F_{1,12}$=0.01), and (F_{71}=0.79, F_{72}=0.05, F_{73}=0, F_{74}=0.05, F_{75}=0.05, F_{76}=0.05, F_{77}=0, F_{78}=0, F_{79}=0, $F_{7,10}$=0, $F_{7,11}$=0, $F_{7,12}$=0). Notice that a simple interpolation is proposed for $y \equiv F_{13}$ because no analytical solution has been found.	

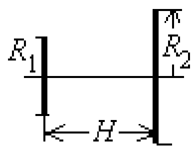
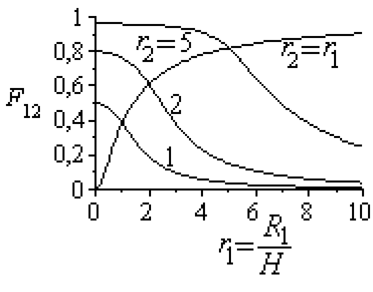
Equal rectangular plates

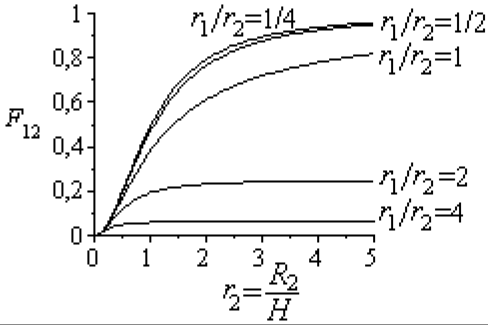
Case	View factor	Plot
Between parallel equal rectangular plates of size $W_1 \cdot W_2$ separated a distance H, with $x=W_1/H$ and $y=W_2/H$. 	$F_{12} = \frac{1}{\pi xy}\left[\ln\frac{x_1^2 y_1^2}{x_1^2+y_1^2-1} + 2x\left(y_1\arctan\frac{x}{y_1}-\arctan x\right) + 2y\left(x_1\arctan\frac{y}{x_1}-\arctan y\right)\right]$ with $x_1 \equiv \sqrt{1+x^2}$ and $y_1 \equiv \sqrt{1+y^2}$ (e.g. for x=y=1, F_{12}=0.1998)	

Equal discs

Case	View factor	Plot
Between two identical coaxial discs of radius R and separation H, with $r=R/H$. 	$F_{12} = 1 + \frac{1-\sqrt{4r^2+1}}{2r^2}$ (e.g. for r=1, F_{12}=0.382)	

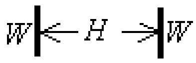
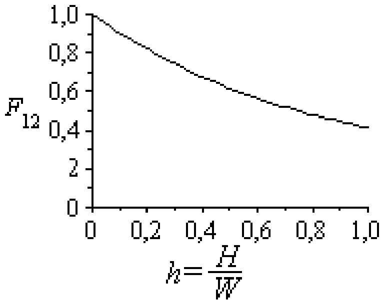
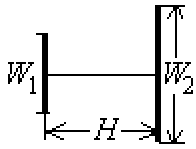
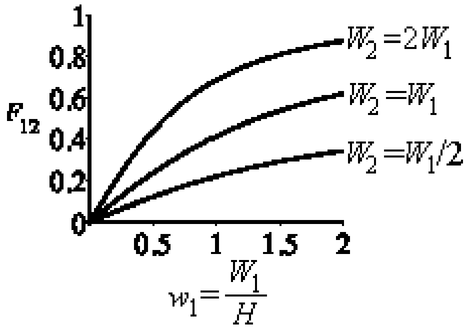
Unequal discs

Case	View factor	Plot
From a disc of radius R_1 to a coaxial parallel disc of radius R_2 at separation H, with $r_1=R_1/H$ and $r_2=R_2/H$. 	$F_{12} = \frac{x-y}{2}$ with $x = 1+1/r_1^2 + r_2^2/r_1^2$ and $y = \sqrt{x^2-4r_2^2/r_1^2}$ (e.g. for r_1=r_2=1, F_{12}=0.382)	

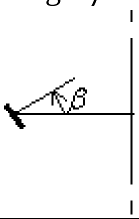
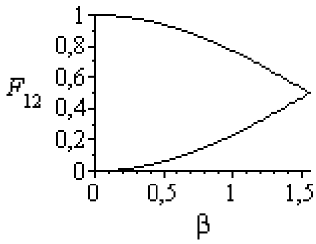
		

Strip to strip

Note. See the crossing-string method, above, for these and other geometries.

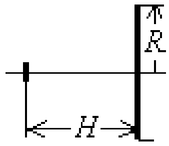
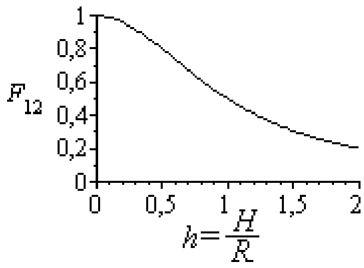
Case	View factor	Plot
Between two identical parallel strips of width W and separation H, with $h=H/W$. 	$F_{12}=\sqrt{1+h^2}-h$ (e.g. for h=1, F_{12}=0.414)	
Between two unequal parallel strips of width W_1 and W_2, and separation H, with $w_1=W_1/H$ and $w_2=W_2/H$. 	$F_{12}=\frac{\sqrt{(w_1+w_2)^2+4}}{2w_1}-\frac{\sqrt{(w_2-w_1)^2+4}}{2w_1}$ (e.g. for $w_1=w_2$=1, F_{12}=0.414)	

Patch to infinite plate

Case	View factor	Plot
From a finite planar plate at a distance H to an infinite plane, tilted an angle β. 	Front side: $F_{12}=\frac{1+\cos\beta}{2}$ Back side: $F_{12}=\frac{1-\cos\beta}{2}$ (e.g. for $\beta=\pi/4$ (45º), $F_{12,front}$=0.854, $F_{12,back}$=0.146)	

Patch to disc

Case	View factor	Plot

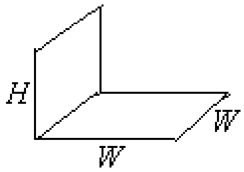
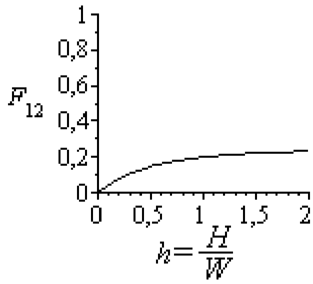
From a patch to a parallel and concentric disc of radius *R* at distance *H*, with *h*=*H*/*R*. 	$$F_{12}=\frac{1}{1+h^2}$$ (e.g. for *h*=1, F_{12}=0.5)	

Perpendicular configurations

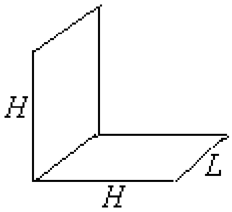
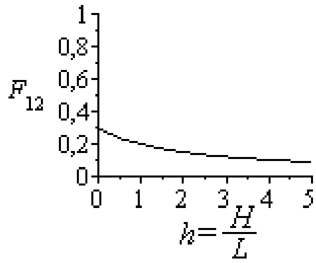
Cylindrical rod to coaxial disc at tone end

(See it under ‘Cylinders’.)

Square plate to rectangular plate

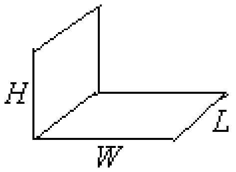
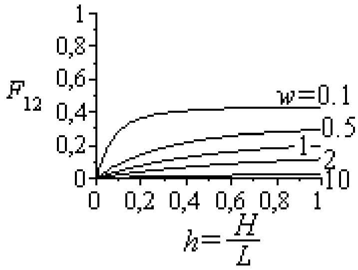
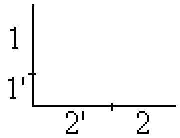
Case	View factor	Plot
From a square plate of with *W* to an adjacent rectangles at 90º, of height *H*, with *h*=*H/W*. 	$$F_{12}=\frac{1}{4}+\frac{1}{\pi}\left[h\arctan\frac{1}{h}-h_1\arctan\frac{1}{h_1}-\frac{h^2}{4}\ln h_2\right]$$ with $h_1=\sqrt{1+h^2}$ and $h_2=\frac{h_1^4}{h^2\left(2+h^2\right)}$ (e.g. for *h*=→∞, F_{12}=→1/4, for *h*=1, F_{12}=0.20004, for *h*=1/2, F_{12}=0.146)	

Rectangular plate to equal rectangular plate

Case	View factor	Plot
Between adjacent equal rectangles at 90º, of height *H* and width *L*, with *h*=*H/L*. 	$$F_{12}=\frac{1}{\pi}\left[2\arctan\left(\frac{1}{h}\right)-\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}h}\right)+\frac{1}{4h}\ln\left(\frac{h_1h_2}{4}\right)\right]$$ with $h_1=2\left(1+h^2\right)$ and $h_2=\left(1-\frac{1}{h_1}\right)^{2h^2-1}$ (e.g. for *h*=1, F_{12}=0.20004)	

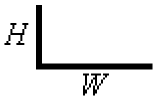
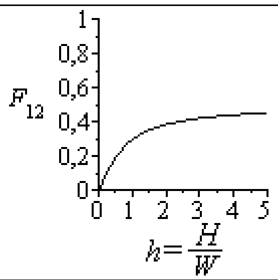
Rectangular plate to unequal rectangular plate

Case	View factor	Plot

From a horizontal rectangle of $W \cdot L$ to adjacent vertical rectangle of $H \cdot L$, with $h=H/L$ and $w=W/L$. 	$F_{12}=\frac{1}{\pi w}\left[h\arctan\left(\frac{1}{h}\right)+w\arctan\left(\frac{1}{w}\right)-\sqrt{h^2+w^2}\arctan\left(\frac{1}{\sqrt{h^2+w^2}}\right)+\frac{1}{4}\ln\left(ab^{w^2}c^{h^2}\right)\right]$ with $a=\frac{(1+h^2)(1+w^2)}{1+h^2+w^2}$, $b=\frac{w^2(1+h^2+w^2)}{(1+w^2)(h^2+w^2)}$, $c=\frac{h^2(1+h^2+w^2)}{(1+h^2)(h^2+w^2)}$ (e.g. for $h=w=1$, $F_{12}=0.20004$)	
From non-adjacent rectangles, the solution can be found with view-factor algebra as shown here 	$F_{1\to2}=F_{1\to2+2'}-F_{1\to2'}=\frac{A_{2+2'}}{A_1}F_{2+2'\to1}-\frac{A_{2'}}{A_1}F_{2'\to1}=$ $=\frac{A_{2+2'}}{A_1}\left(F_{2+2'\to1+1'}-F_{2+2'\to1'}\right)-\frac{A_{2'}}{A_1}\left(F_{2'\to1+1'}-F_{2'\to1'}\right)$	

Strip to strip

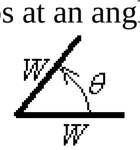
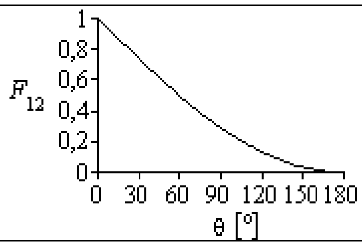
Note. See the crossing-string method, above, for these and other geometries.

Case	View factor	Plot
Adjacent long strips at 90º, the first (1) of width W and the second (2) of width H, with $h=H/W$. 	$F_{12}=\frac{1+h-\sqrt{1+h^2}}{2}$ (e.g. $F_{12}\big\|_{H=W}=1-\frac{\sqrt{2}}{2}=0.293$)	

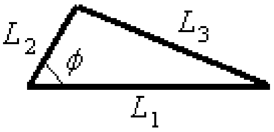
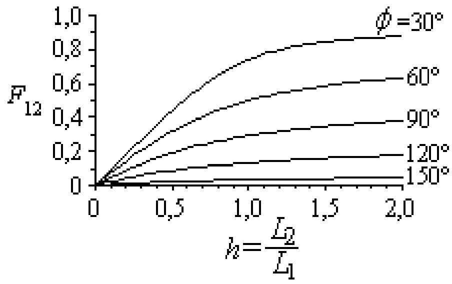
Tilted strip configurations

Note. See the crossing-string method, above, for these and other geometries.

Equal adjacent strips

Case	View factor	Plot
Adjacent equal long strips at an angle α. 	$F_{12}=1-\sin\frac{\alpha}{2}$ (e.g. $F_{12}\big\|_{\frac{\pi}{2}}=1-\frac{\sqrt{2}}{2}=0.293$)	

Triangular prism

Case	View factor	Plot
Between two sides, 1 and 2, of an infinite long triangular prism of sides L_1, L_2 and L_3 , with $h=L_2/L_1$ and ϕ being the angle between sides 1 and 2. 	$F_{12}=\frac{L_1+L_2-L_3}{2L_1}=$ $=\frac{1+h-\sqrt{1+h^2-2h\cos\phi}}{2}$ (e.g. for h=1 and $\phi=\pi/2$, F_{12}=0.293)	

NUMERICAL COMPUTATION

Several numerical methods may be applied to compute view factors, i.e. to perform the integration implied in (2) from the general expression (1). Perhaps the simpler to program is the random estimation (Monte Carlo method), where the integrand in (2) is evaluated at N random quadruples, (c_{i1}, c_{i2}, c_{i3}, c_{i4}) for i=1..N, where a coordinates pair (e.g. c_{i1}, c_{i2}) refer to a point in one of the surfaces, and the other pair (c_{i3}, c_{i4}) to a point in the other surface. The view factor F_{12} from surface A_1 to surface A_2 is approximated by:

$$F_{12}=\frac{A_2}{N}\sum_{i=1}^{N}\left.\frac{\cos\beta_1\cos\beta_2}{\pi r_{12}^2}\right|_i \tag{11}$$

where the argument in the sum is evaluated at each ray i of coordinates (c_{i1}, c_{i2}, c_{i3}, c_{i4}).

Example 4. Compute the view factor from vertical rectangle of height H=0.1 m and depth L=0.8 m, towards an adjacent horizontal rectangle of W=0.4 m width and the same depth. Use the Monte Carlo method, and compare with the analytical result.

Sol. The analytical result is obtained from the compilation above for the case of 'With plates and discs / Perpendicular configurations / Rectangular plate to unequal rectangular plate', obtaining, for $h=H/L$=0.1/0.8=0.125 and $w=W/L$=0.4/0.8=0.5 the analytical value F_{12}=0.4014 (mind that we want the view factor from the vertical to the horizontal plate, and what is compiled is the opposite, so that a reciprocity relation is to be applied).

For the numerical computation, we start by setting the argument of the sum in (11) explicitly in terms of the coordinates (c_{i1}, c_{i2}, c_{i3}, c_{i4}) to be used; in our case, Cartesian coordinates (x_i, y_i, z_i, y'_i) such that (x_i, y_i) define a point in surface 1, and (z_i, y'_i) a point in surface 2. With that choice, $\cos\beta_1=z/r_{12}$, $\cos\beta_2=x/r_{12}$, and $r_{12}=\sqrt{x^2+z^2+\left(y_2-y_1\right)^2}$, so that:

$$F_{12} = \frac{A_2}{N}\sum_{i=1}^{N}\frac{\cos\beta_1\cos\beta_2}{\pi r_{12}^2}\bigg|_i = \frac{WL}{N}\sum_{i=1}^{N}\frac{zx}{\pi r_{12}^4}\bigg|_i = \frac{WL}{N}\sum_{i=1}^{N}\frac{zx}{\pi\left[x^2+z^2+\left(y-y'\right)^2\right]}\Bigg|_i = \frac{WL}{N}\sum_{i=1}^{N} f_i$$

where f_i is the value of the function at a random quadruple (x_i, y_i, z_i, y'_i). A Matlab coding may be:

```
W=0.4; L=0.8; H=0.1; N=1024;                            %Data, and number of rays to be used
f= @(z,y1,x,y2) (1/pi)*x.*z./(x.^2+z.^2+(y2-y1).^2).^2;  %Defines the function
for i=1:N fi(i)=f(rand*H, rand*L, rand*W, rand*L);end;   %Computes its values
F12=(W*L/N)*sum(fi)                                      %View factor estimation
```

Running this code three times (it takes about 0.01 s in a PC, for N=1024), one may obtain for F_{12} the three values 0.36, 0.42, and 0.70, but increasing N increases accuracy, as shown in Fig. E4.

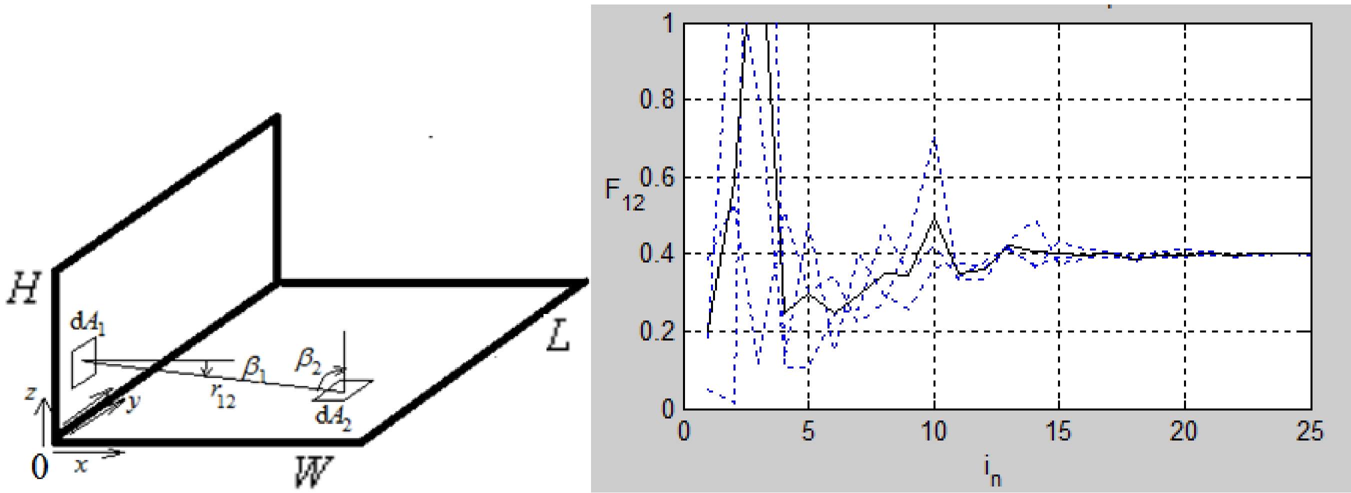


Fig. E4 Geometry for this example (with notation used), and results of the F_{12}-computation with a number $N=2^{i_n}$ of random quadruplets (e.g. $N=2^{10}=1024$ for $i_n=10$); three runs are plotted, with the mean in black.

REFERENCES

Howell, J.R., "A catalog of radiation configuration factors", McGraw-Hill, 1982. (web.)
Siegel, R., Howell, J.R., Thermal Radiation Heat Transfer, Taylor & Francis, 2002.

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